Math 433/833 **Assignment # 0** **12 Jan 11**

Due: Jan 19; Give each problem at most 10 minutes; Full credit for submission.

1. Find the maximum and minimum of $f(x) = \dfrac{x(x-1)}{x^2+3x+3}$ on the interval $[0,6]$ and then on the real line.

2. Compute the gradient $\nabla f(\mathbf{x})$ of the function $f(x_1, x_2, x_3) = 3x_1^2 + 4x_1x_2 + x_2\sin(x_3)$.

3. Solve the following linear system of equations. If the system is consistent, write the solution in vector form.
$$\left\{\begin{array}{rrrcr} x & -y & +z & = & 0 \\ -x & +3y & +z & = & 5 \\ 3x & +y & +7z & = & 10 \end{array}\right.$$

4. For each matrix below, determine if it is invertible. If so, compute its inverse.
$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}.$$

5. Let
$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}.$$
Find all of the eigenvalues of A and bases for the corresponding eigenspaces.

6. Are the following vectors linearly independent? Prove your answer.
$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$