Math 308 - Section 506 - Project 1

Instructor: Dr. Adam Larios

Friday, 27 Jan 2012

Due date: Friday, 10 Feb 2012

Instructions. Submit one report per group, with the names of all the group members included. While there are no specific neatness requirements, your work should look professional, and you should submit it in a form that would be appropriate for submitting to a boss at a job you care about. Credit will be given for full, complete, and thoughtful analysis; however, a good report is not necessarily a long report. Do not submit fluff or filler, but only quality work that you are proud of.

Submit all of your code, along with any relevant graphs, mathematical calculations, analysis, and explanations. Make sure graphs are well labeled and referenced, and that they are easy to read. Please note that each person is responsible for 100% of the work, so it is your responsibility to keep all group members on task, or to finish the work yourself.

Part I.

1. Consider the differential equation
$$y' = e^{y-1} - 2y$$
Estimate the equilibrium solutions by plotting the direction field.

2. Now consider the equation
$$y' = e^{y-a} - 2y$$
where a is a constant. How does the value of a influence the location and number of the equilibrium solutions? Be sure to test many values of a. Plot a few key examples demonstrating your findings.

3. Finally, consider
$$y' = e^{y-1} - 2y^n$$
where n is a positive integer. Describe how n affects the behavior of the integral curves for the equation. Plot at least three examples with several integral curves.

Part II.

Suppose a certain number of deer are introduced to a very large island which is uninhabited by predators. Neglecting any constraints, suppose their population increases at a rate of 34% per year.

1. We saw in class how to write down a model which took a "carrying capacity" into consideration. Suppose the carrying capacity for the deer on this island is 6000 deer. Write down a differential equation that models the population, and solve it numerically for several different initial values of deer introduced.

2. Now suppose that, in addition to accounting for the carrying capacity, we also account for the population becoming too sparse. That is, if the population drops below a certain number, it starts to decrease. Write down a differential equation describing this situation, and solve your equation numerically. What are the possible long-term outcomes for the deer population?

3. Compare your models with the situation described in the webcomic in the link below. (**Don't look at it until after you have done the previous two problems! Otherwise, you will ruin the surprise for yourself!**)

   `http://www.recombinantrecords.net/2011/02/09/st-matthew-island/`

   What are the differences between your predictions and what happened at St. Matthew Island? What factors does your model not take into account? How could your model be improved to more accurately model reality?