HOMEWORK 1
MATRIX ALGEBRA AND DIRECT METHODS FOR LINEAR SYSTEMS

DUE: WEDNESDAY, 4 SEPTEMBER 2013

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Solve any set of 5 problems, worth 20 points each. Indicate which 5 problems you would like graded. Fully justify your work.

(1) (Problem 1a, p. 158 in the textbook) Prove that the inverse of a nonsingular upper-triangular matrix is also upper-triangular.

(2) (Problem 12, p. 148 in the textbook) Let A be an invertible $n \times n$ matrix, and let u and v be two vectors in $\mathbb{R}^n$. Consider the $(n+1) \times (n+1)$ matrix M given by

$$M := \begin{pmatrix} A & u \\ v^T & 0 \end{pmatrix}$$

   (a) Find necessary and sufficient conditions on u and v that make M invertible.

   (b) Give a formula for M^{-1} when it exists.

(3) (Problem 8b, p. 159 in the textbook) Write the **column version** of the Doolittle algorithm, which computed the k^{th} column of L and the k^{th} column of U at the k^{th} step. (Consequently, the order of computing is $u_{1k},\ u_{2k},\ \dots,\ u_{kk},\ \ell_{k+1,k},\ \dots,\ \ell_n k$ at the k^{th} step.) Count the number of arithmetic operations (counting the additions/subtractions and multiplications/divisions separately).

(4) (Problem 26, p. 160 in the textbook) Prove: A is positive definite and B is nonsingular if and only if BAB^T is positive definite.

(5) (Problem 27, p. 160 in the textbook) If A is positive definite and invertible, does it follow that A^{-1} is also positive definite? Prove or give a counter-example.

(6) (Problem 16, p. 148 in the textbook) For what values of a is the following matrix positive definite?

$$A = \begin{pmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{pmatrix}$$

(7) Discuss the positive-definiteness of the following the matrices.

$$A = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 2 & 5 & 2 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & 1 & 0 & 3 \end{pmatrix},\ B = \begin{pmatrix} 3 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix},\ C = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$