LAST NAME: Key FIRST NAME: ______________ Quiz 1

MATH 602, Differential Equations Prof: Dr. Adam Larios

Notes, books, and calculators are not authorized. Show all your work in the blank space you are given. Always justify your answer. Answers without adequate justification will not receive credit.

1. Let $\varphi(x,y) = e^x \sin(y) + \cos(1+xy)$. Compute $\partial_x \varphi(x,y)$ and $\partial_y \varphi(x,y)$. (Do not try to simplify the results).

$$\partial_x \varphi = e^x \sin y + \cos(1+xy)\cdot y$$

$$\partial_y \varphi = e^x \cos(y) + \cos(1+xy)\cdot x$$

2. Consider the following heat equation in variables $x \in [a,b]$ and $t > 0$.

$$\begin{cases} \partial_t T - k\partial_{xx} T = f(x), \\ T(a) = 0, \\ -k\partial_n T(b) = 0. \end{cases}$$

Let us assume $k > 0$ is a constant, and $f(x) = kx^2$. Compute the steady state solution (i.e., where $\partial_t T = 0$). (Here, ∂_n means the derivative in the normal or "outward pointing" direction.)

Steady state means $\partial_t T = 0$, so we solve

$$\begin{cases} -\cancel{k}\partial_{xx} T = \cancel{k} x^2 \\ T(a) = 0 \\ -kT_x(b) = 0 \end{cases} \leftarrow \partial_n T = \frac{\partial T}{\partial x}\cdot n$$

. Since b is on the right, $n = 1$. (At a, $n = -1$).

Integrate twice and use fundamental theorem of calculus:

$$\Rightarrow T' = \frac{1}{3}x^3 + C_1 \qquad 0 = T'(b) = \frac{1}{3}b^3 + C_1 \Rightarrow C_1 = -\frac{1}{3}b^3$$

$$\Rightarrow T(x) = \frac{1}{3}\cdot\frac{1}{4}x^4 + C_1 x + C_2 \qquad 0 = T(a) = \frac{1}{12}a^4 + C_1 a + C_2$$

Thus, $T(x) = \frac{1}{12}x^4 - \frac{1}{3}b^3 x - \overbrace{\left(\frac{1}{12}a^4 - \frac{1}{3}b^3 a\right)}^{c_2}$